CEE Growth & Development

Solving Solow Model

Michælmas 2013

World Development Report 2014

- Schebeck Palace Room 8

Solow Model: Basics

Assumption 1. Production Function

$$Y = AK^{\alpha}L^{1-\alpha}$$

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Assumption 1. Production Function

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Assumption 3. Population Growth Rate

$$\frac{\dot{L}}{L} = n$$

Solow Model: Transformations

1. Production Function

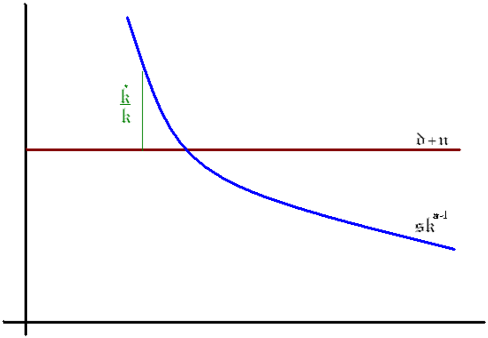
$$\frac{Y}{L} = \frac{AK^{\alpha}L^{1-\alpha}}{L^{\alpha}L^{1-\alpha}} = Ak^{\alpha}$$

2. Fundamental LoM

$$\begin{aligned} \frac{\dot{K}}{K} &= s\frac{Y}{K} - \delta \\ \frac{\dot{k}}{k} + n &= s\frac{y}{k} - \delta \\ \dot{k} &= sAk^{\alpha} - (\delta + n)\,k \end{aligned}$$

Solow Model: Transformations

$$\frac{\dot{k}}{k} = sAk^{\alpha-1} - (\delta + n)$$



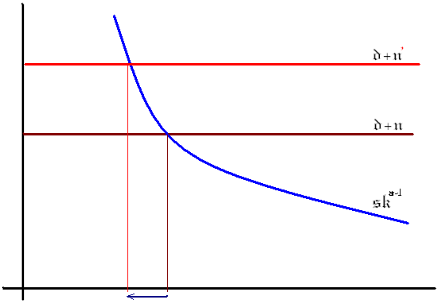
Solow Model: Solving for SS

$$
\begin{aligned}
\left.\frac{\dot{k}}{k}\right|_{SS} &= 0 \\
sAk^{\alpha-1} - (\delta + n) &= 0 \\
k|_{SS} &= \left(\frac{sA}{\delta + n}\right)^{\frac{1}{1-\alpha}}
\end{aligned}
$$

$$
y|_{SS} = A\left(\frac{sA}{\delta + n}\right)^{\frac{\alpha}{1-\alpha}}
$$

Capital Dilution

$$k_{SS} = \left(\frac{s}{\delta + n}\right)^{\frac{1}{1-\alpha}}$$



Solow Model: Aggregates on the Balanced Growth Path

$$\left.\frac{\dot{k}}{k}\right|_{ss} = 0$$

$$\begin{aligned}
\frac{\dot{k}}{k} &= 0 \Longrightarrow \frac{\dot{y}}{y} = 0 \\
\frac{\dot{k}}{k} &= 0 \Longrightarrow \frac{\dot{K}}{K} = n \\
\frac{\dot{k}}{k} &= 0 \Longrightarrow \frac{\dot{Y}}{Y} = n
\end{aligned}$$

Solow Model Predictions

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- In their steady states the aggregates will grow (with rate of the population growth), i.e. aggregates are on the BGP, while the per capita variables are constant
- The larger the population growth rate, the smaller the steady-state values of per capita variables are:

$$\frac{y_i^{ss}}{y_j^{ss}} = \left(\frac{n_j + \delta}{n_i + \delta}\right)^{\frac{\alpha}{1-\alpha}}$$

Lucas (1990)

Change the main question slightly

- Why are some countries poor while the others are rich?
- Why doesn't capital flow from rich to poor countries?

Lucas (1990)

Why Doesn't Capital Flow from Rich to Poor Countries?

$$\pi = AK^{\alpha}L^{1-\alpha} - rK - wL$$

$$\begin{aligned} AK^{\alpha-1}L^{1-\alpha} - r &= 0 \\ Ak^{\alpha-1} &= r \end{aligned}$$

That is

$$\uparrow r \overset{\alpha\in(0,1)}{\Longrightarrow} \downarrow k \overset{y=Ak^{\alpha}}{\Longrightarrow} \downarrow y$$

Lucas (1990)
Why Doesn't Capital Flow from Rich to Poor Countries?

$$\begin{cases} r = Ak^{\alpha-1} \\ y = Ak^{\alpha} \end{cases} \implies y = \frac{A^{\frac{1}{1-\alpha}}}{r^{\frac{\alpha}{1-\alpha}}}$$

That is

$$\frac{y^{rich}}{y^{poor}} = \left(\frac{A^{rich}}{A^{poor}}\right)^{\frac{1}{1-\alpha}} \left(\frac{r^{poor}}{r^{rich}}\right)^{\frac{\alpha}{1-\alpha}}$$

Problem

Assume $A^{NO} = A^{MD} = 1$ and $\alpha = 1/3$. $GDPpc^{Norway} = \$50000$ and $GDPpc^{Moldova} = \$1000$. Why Doesn't Capital Flow from Rich to Poor Countries?

Human Capital

$$Y = AK^{\alpha}\left(hL\right)^{1-\alpha}$$

$$\frac{y_i}{y_i} = \frac{h_i A^{\frac{1}{1-\alpha}}\left(\frac{s}{n+\delta}\right)^{\frac{\alpha}{1-\alpha}}}{h_j A^{\frac{1}{1-\alpha}}\left(\frac{s}{n+\delta}\right)^{\frac{\alpha}{1-\alpha}}} = \frac{h_i}{h_j}$$

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- Human capital
- Education

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- Human capital
- Education
- Experience
- Health, and the like

Your paper

Need to have a research question:

`General question (yes/no)`

vs.

`Special questions (wh-words)`

Some logical fallacies

- Selection bias
- Post hoc ergo propter hoc
- Cum hoc ergo propter hoc
- Lord Russel's Chicken and the Black Swan Effect
- Jeanne d'Arc Syndrome
- ...